TRANSPORT OF RADIATION ENERGY IN MATERIALS WITH VARIABLE OPTICAL PROPERTIES

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A solution is presented for the problem of the distribution of absorbed energy and of the fluxes of monochromatic and integrated radiation in actual absorbing and scattering materials with optical properties which vary along a coordinate.

Studies of the transport of energy of monochromatic radiation in materials with optical properties varying along a coordinate are becoming of great practical importance [1-6, 8-18]. Some particular problems have been considered [1-4, 6, 8-18] which involve monochromatic radiation energy transport in the atmosphere and in other turbid media with coefficients of absorption [8, 10] and emission [17] that vary along a coordinate. Attempts have been made to include the effect on light scattering of coordinate-dependent changes in the scattering kernel [6, 8, 11-13], in the concentration of scattering particles [4, 16], in the optical anisotropy of nonuniform media [1, 15], and in the dependence of the coefficients of absorption [1, 2, 4, 9, 15] and scattering [2] on the density of the incident flux. In many papers, the scattering coefficient is assumed constant over the thickness of a layer.

The creation of effective methods for calculating the transport of the energy of integrated radiation in absorbing and scattering materials is also important for practical application in various fields of science and engineering. In the propagation of integrated fluxes of solar and other infrared radiation in materials, the spatial distribution and spectral composition of the radiation energy varies along a coordinate because of the effect of multiple scattering by optical nonuniformities, which results in a variation in the values of the mean integral coefficients of absorption and scattering [5].

It is practical to solve the general problem of energy transport for monochromatic and integrated radiation in materials with optical properties varying along a coordinate by means of a refined differential-difference method. This method offers an opportunity to obtain rather simple relationships for the attenuation of the radiation flux in various materials [5].

The optical properties of materials exposed to solar radiation and to infrared radiation in thermoradiation devices do not depend on the density of the incident flux but are determined by the physical and chemical properties of the materials and their changes during irradiation. Furthermore, both coefficients characterizing the optical properties of a material capable of absorbing and scattering radiation vary over the thickness of a layer.

A solution is presented here for the general problem of bilateral irradiation of a plane layer of material, for which the dependence of optical properties on a coordinate may be the result of the following: variation along a coordinate of the values of the spectral absorption coefficient $k_\lambda(x)$, spectral scattering coefficient $\sigma_\lambda(x)$, and the scattering kernel $\chi_\lambda(x)$, as well as variations of the spatial distribution and spectral composition of the radiation flux.

In the general case, the optical properties of a layer as a function of the coordinate x are characterized by the spectral absorption coefficient $\bar{k}_\lambda(x)$ and the "backward" scattering coefficient $s_\lambda(x)$ averaged over a half-space, which convey cumulative information about the spatial distribution of radiation energy at the depth x, about the optical properties of the material, and about the relations

$$\bar{k}_\lambda(x) = m_\lambda(x)\, k_\lambda(x); \quad s_\lambda(x) = m_\lambda(x)\, \delta_{s,\lambda}(x)\, \sigma_\lambda(x), \tag{1}$$

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which are associated with the fundamental characteristics for absorption and scattering $k_\lambda(x)$ and $\sigma_\lambda(x)$.

The coefficients m_λ and $\delta_{s,\lambda}$ also depend on x and take into account the irradiation conditions for an elementary layer of thickness dx and the form of the scattering kernel. These quantities can take on the following values [5]: $m_\lambda = 1$ for directional irradiation, $m_\lambda = 2$ for completely diffuse irradiation, and $m_\lambda \gtrless 2$ for incompletely diffuse irradiation; $\delta_{s,\lambda} = 0.5$ for a symmetric scattering kernel, $\delta_{s,\lambda} < 0.5$ for forward elongation, and $\delta_{s,\lambda} \leq 1$ for backward elongation.

In this case, we obtain a system of linear differential equations of first order with two variable coefficients $\bar{k}_\lambda(x)$ and $s_\lambda(x)$ with respect to the densities of the opposing monochromatic fluxes E_λ^+ and E_λ^- similar to that found in [2,4,5].

By simple transformations, replacing the variable x by the optical depth

$$\tau = \int_0^x [\bar{k}_\lambda(x) + s_\lambda(x)]\, dx, \tag{2}$$

the layer thickness l by the optical thickness τ_l determined from Eq. (2) when x = 1, and introducing the Schuster criterion $\Lambda_{e,\lambda}$ [5] which represents the effective probability of photon survival in an elementary scattering event,

$$\Lambda_{e,\lambda}(x) = \frac{s_\lambda(x)}{\bar{k}_\lambda(x) + s_\lambda(x)} = \frac{\delta_{s,\lambda}(x)\Lambda_\lambda(x)}{1-[1-\delta_{s,\lambda}(x)]\Lambda_\lambda(x)}, \tag{3}$$

we obtain a new system of linear differential equations with the single variable coefficient $\Lambda_{e,\lambda}(\tau)$ with respect to the density of the resultant flux $q_\lambda = E_\lambda^+ - E_\lambda^-$ and the spatial irradiance $E_{0,\lambda} = E_\lambda^+ + E_\lambda^-$,

$$\frac{dq_\lambda}{d\tau} = -[1-\Lambda_{e,\lambda}(\tau)]E_{0,\lambda}; \qquad \frac{dE_{0,\lambda}}{d\tau} = [1+\Lambda_{e,\lambda}(\tau)]\, q_\lambda. \tag{4}$$

The boundary conditions will be: at $\tau = 0$,

$$E_{0,\lambda}(0) = (1+R_\lambda)E_{1,\lambda} + T_\lambda E_{2,\lambda}, \quad q_\lambda(0) = (1-R_\lambda)E_{1,\lambda} - T_\lambda E_{2,\lambda}, \tag{5}$$

and at $\tau = \tau_l$,

$$E_{0,\lambda}(\tau_l) = T_\lambda E_{1,\lambda} + (1+R_\lambda)E_{2,\lambda}, \quad q_\lambda(\tau_l) = T_\lambda E_{1,\lambda} - (1-R_\lambda)E_{2,\lambda}. \tag{6}$$

The solution of the system (4) with respect to q_λ or $E_{0,\lambda}$ for an arbitrary function $\Lambda_{e,\lambda}(\tau)$ leads to a self-adjoint differential equation of second order:

$$\frac{d}{d\tau}\left[\frac{1}{1-\Lambda_{e,\lambda}(\tau)}\frac{dq_\lambda}{d\tau}\right] - [1+\Lambda_{e,\lambda}(\tau)]\, q_\lambda = 0, \tag{7}$$

the solution of which can only be obtained in explicit form for a known function $\Lambda_{e,\lambda}(\tau)$.

The dependence on optical depth τ of the mean effective photon survival probability $\Lambda_{e,\lambda}$ in many actual processes of monochromatic radiation energy transport is closely approximated for unilateral irradiation by the following function:

$$\Lambda_{e,\lambda}(\tau) = 1 - \frac{a}{\tau + b}. \tag{8}$$

The constants a and b have the sense of dimensionless values of the averaged coefficients of absorption and extinction for a layer of unit thickness at $\tau = 0$, $a = \bar{k}_\lambda^*(0)$, and $b = \varepsilon_{e,\lambda}^*(0)$:

$$\Lambda_{e,\lambda}(0) = 1 - \frac{\bar{k}_\lambda^*(0)}{\varepsilon_{e,\lambda}^*(0)} = 1 - \frac{\bar{k}_\lambda^*(0)}{\bar{k}_\lambda^*(0) + s_\lambda^*(0)}. \tag{9}$$

Such a rise in the value of the mean effective photon survival probability follows from the fact that the density of the surface layer of the coatings of actual materials of plant and animal origin decreases rapidly with depth and the scattering and absorption coefficients fall in direct proportion to density according to experimental data [5]. Furthermore, during infrared irradiation of various porous capillary materials and food products, a denser crusty layer is also formed which has a density which decreases with depth. Therefore, $\Lambda_{e,\lambda}$ increases as τ increases, since the absorption coefficient decreases.

The quantities m_λ and $\delta_{s,\lambda}$, which take into account irradiation conditions and the form of the scattering kernel, also vary with depth within a layer during irradiation by directional and incompletely diffuse fluxes. These quantities also vary comparatively rapidly in the surface layer [3], since a "deep mode,"

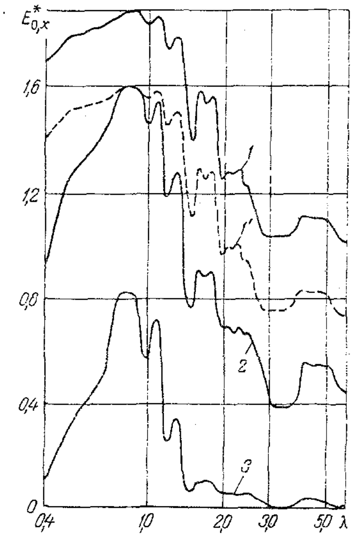

Fig. 1. Dependence of spatial irradiance $E^*_{0,x}$ on wavelength λ (μm) in a semiinfinite layer of starch at various depths x (mm): 1) x = 0; 2) x = 0.2; 3) x = 1.0; 1') curve 1 translated to match with curve 2.

where the irradiation conditions for an elementary layer are close to ideally diffuse and the quantities m_λ and $\delta_{s,\lambda}$ are constant, begins at large τ in absorbing and scattering materials.

For the materials specified, the complex dependence of $\Lambda_{e,\lambda}$ on τ in the general case of bilateral irradiation of a flat layer can be represented in the form

$$\Lambda_{e,\lambda}(\tau) = 1 - \frac{a}{\tau+b} - \frac{a}{c-\tau}, \tag{10}$$

where a, b, and c are constants which depend on the optical properties of the material and on the spatial distribution of the incident monochromatic radiation flux densities $E_{1,\lambda}$ and $E_{2,\lambda}$.

Using Eq. (10), we transform Eq. (7) into a Bessel equation [7] by replacing τ with the new variable $\xi(\tau, \Lambda_{e,\lambda})$:

$$\frac{d^2q_\lambda}{d\xi^2} + \frac{1-2\varkappa}{\xi}\frac{dq_\lambda}{d\xi} + \left(\beta^2\gamma^2\xi^{2\gamma-2} + \frac{\alpha^2-\nu^2\gamma^2}{\xi^2}\right)q_\lambda = 0. \tag{11}$$

The solution of Eq. (11) is a cylinder function of the form [7]

$$q_\lambda(\xi) = \xi^\varkappa Z_\nu(\beta\xi^\gamma), \tag{12}$$

which is expressed through modified Bessel functions of the first and second kind of order ν – $J_\nu(Z)$ and $K_\nu(Z)$.

The general solution (12) found provides an opportunity to determine analytic expressions for the resultant flux density $q_\lambda(\tau)$ and the spatial irradiance $E_{0,\lambda}(\tau)$ of monochromatic radiation in a layer of material with optical properties that vary with thickness for a given function $\Lambda_{e,\lambda}(\tau)$.

Using Eqs. (4) and (10), the general solution (12) takes the form

$$q_\nu(\tau) = Z_\nu(\beta\xi^\nu) = Z_{2ai}\left[2\sqrt{\frac{2a}{b+c}}\, i\sqrt{(\tau+b)(c-\tau)}\right] \tag{13}$$

for bilateral irradiation and can be expressed through the Bessel functions $J_\nu(Z)$ and $K_\nu(Z)$.

For a whole series of actual materials in the third and fourth classes with respect to optical properties [5] that are sufficiently strong scatterers ($R_{\infty,\lambda} > 0.5$, $\Lambda_{e,\lambda} > 0.8$), Eq. (13) can be limited to the first terms in the expansion in a series of the Bessel functions $J_\nu(Z)$ and $K_\nu(Z)$ (for large Z [7]). Then the general solution for bilateral irradiation takes the rather simple form

$$q_\lambda(\tau) = \left[C_1\exp\left(-2\sqrt{\frac{2a}{b+c}\xi}\right) + C_2\exp\left(2\sqrt{\frac{2a}{b+c}\xi}\right)\right]\xi^{-\frac{1}{4}}, \tag{14}$$

where $\xi = (\tau + b)(c - \tau)$.

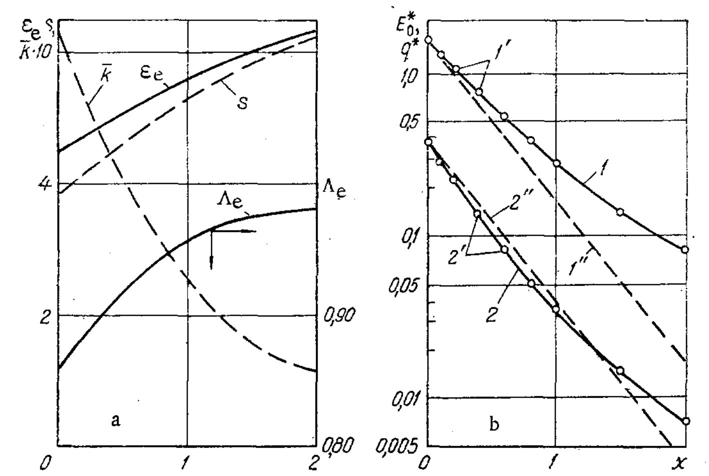

Fig. 2. Variation of integral optical characteristics (a) and radiation field (b) with depth x (mm) in potato starch during irradiation with KG-1000 lamps: 1,1',1") spatial irradiance E_0^*; 2,2',2") resultant flux density q^*; 1,2) from Eqs. (24) and (25); 1',2') from numerical integration [5]; 1",2") by the method of averaged characteristics [5]. s, ε_e, $\bar{k}\cdot 10$, mm^{-1}.

The constants of integration C_1 and C_2 in Eq. (14) are determined from the boundary conditions (5) and (6).

The amount of radiation energy absorbed per unit time by an elementary volume at the depth x is determined from the law of conservation of energy [4,5,8] with Eq. (4) taken into consideration:

$$w_\lambda(\tau) = -\frac{dq_\lambda}{d\tau} = [1 - \Lambda_{e,\lambda}(\tau)]\, E_{0,\lambda}(\tau). \tag{15}$$

In the case of unilateral irradiation ($E_{2,\lambda} = 0$) of a layer of finite thickness τ_l of a material with varying optical properties $\Lambda_{e,\lambda}(\tau)$, the quantities q_λ and $E_{0,\lambda}$ for spectral radiation are given by the following expressions:

$$q_\lambda(\tau) = \frac{E_{1,\lambda}(1 - R_{\infty,\lambda})}{1 - B_1\Psi_\lambda^2}\{\exp[-2\sqrt{2ab}\,(\sqrt{\xi} - 1)] + \Psi_\lambda^2 \exp[2\sqrt{2ab}\,(\sqrt{\xi} - 1)]\}\,\xi^{-\frac{1}{4}}, \tag{16}$$

$$E_{0,\lambda}(\tau) = \frac{E_{1,\lambda}(1 + R_{\infty,\lambda})}{1 - B_1\Psi_\lambda^2}\,\frac{1 + 4\sqrt{2ab\xi}}{2a\sqrt[4]{\xi}}\left\{\exp[-2\sqrt{2ab}\,(\sqrt{\xi} - 1)] - \Psi_\lambda^2\frac{4\sqrt{2ab\xi} - 1}{4\sqrt{2ab\xi} + 1}\exp[2\sqrt{2ab}\,(\sqrt{\xi} - 1)]\right\}. \tag{17}$$

The following notation were used in Eqs. (16) and (17):

$$R_{\infty\lambda} = \frac{4(\sqrt{2ab} - a) + 1}{4(\sqrt{2ab} + a) + 1}, \tag{18}$$

$$\Psi_\lambda^2 = \frac{4(\sqrt{2ab\xi_l} - a) + 1}{4(\sqrt{2ab\xi_l} + a) - 1}\exp[-4\sqrt{2ab}\,(\sqrt{\xi_l} - 1)], \tag{19}$$

$$B_1 = \frac{4(\sqrt{2ab} - a) - 1}{4(\sqrt{2ab} + a) + 1}, \tag{20}$$

$$\xi = \sqrt{1 + \frac{\tau}{b}}, \qquad \xi_l = \sqrt{1 + \frac{\tau_l}{b}}. \tag{21}$$

The values of the thermoradiative characteristics of a layer of finite thickness with varying optical properties are determined from Eq. (16) for the unilateral irradiation condition $E_{2,\lambda} = 0$ using Eqs. (5) and (6):

$$R_\lambda = 1 - \frac{q_\lambda(0)}{E_{1,\lambda}} = \frac{R_{\infty,\lambda} - (1 + B_1 - R_{\infty,\lambda})\Psi_\lambda^2}{1 - B_1\Psi_\lambda^2}, \tag{22}$$

$$T_\lambda = \frac{q_\lambda(\tau_l)}{E_{1,\lambda}} = \frac{1 - R_{\infty,\lambda}}{1 - B_1\Psi_\lambda^2} \cdot \frac{8\sqrt{ab\xi_l}\exp[-2\sqrt{2ab}(\sqrt{\xi_l} - 1)]}{4(\sqrt{2ab\xi_l} + a) - 1}. \tag{23}$$

In an optically infinitely thick layer, the resultant flux density and spatial irradiance are found from Eqs. (16) and (17) when $\tau_l \to \infty$:

$$q_\lambda(\tau) = E_{1,\lambda}(1 - R_{\infty,\lambda})\left(1 + \frac{\tau}{b}\right)^{-\frac{1}{4}} \exp\left[-2\sqrt{2ab}\left(\sqrt{1 + \frac{\tau}{b}} - 1\right)\right], \tag{24}$$

$$E_{0,\lambda}(\tau) = E_{1,\lambda}(1 + R_{\infty,\lambda}) \frac{1 + 4\sqrt{2ab\left(1 + \frac{\tau}{b}\right)}}{4a\left(1 + \frac{\tau}{b}\right)^{\frac{1}{4}}} \exp\left[-2\sqrt{2ab}\left(\sqrt{1 + \frac{\tau}{b}} - 1\right)\right]. \tag{25}$$

The general solutions (12)-(15) and (16)-(25) obtained for the problem of monochromatic radiation energy transport in materials with optical properties varying along a coordinate for unilateral and bilateral irradiation are also applicable in the case of material irradiation by an integrated radiation flux through replacement of the spectral $\Lambda_{e,\lambda}(\tau)$ by the integrated $\Lambda_e(\tau)$.

Equations (15)-(25) provide an opportunity for rather simple calculation, without integration of the analytic expressions for $q_\lambda(x)$ and $E_{0,\lambda}(x)$ over the spectrum of the radiator, of the integrated radiation field in selectively absorbing and scattering materials by taking into account the variation over a coordinate of the mean integrated optical characteristics by means of the function $\Lambda_e(\tau)$ and the optical depth τ.

To establish the form of the function $\Lambda_e(\tau)$, we consider an actual process of integrated radiation energy transport in a semiinfinite layer of a typical scattering porous capillary colloidal material — potato starch (W = 11.8%) — irradiated with KG-1000 lamps. From Fig. 1, in which the dependence of the dimensionless magnitude of the spatial irradiance $E_{0,x}^*(\lambda)$ on wavelength is shown for given values of the depth x, it is clear that the spectral composition of the integrated radiation varies with x because of the effects of multiple scattering and selective absorption. Those portions of the radiation for which k_λ is least and σ_λ greatest penetrate to greater depths.

The mean integral absorption and backscattering coefficients $\bar{k}(x)$ and $s(x)$ acting at the depth x are determined by averaging $E_{0,x}(\lambda)$ and $q_x(\lambda)$ over spectral composition at a fixed value of x over the spectral range $\lambda_1 - \lambda_2$ of the incident radiation flux [5]. It is clear from Fig. 2a that the mean effective photon survival probability Λ_e increases with depth and approaches one at large x (pure scattering). The coefficients $\varepsilon_e(x)$ and $s(x)$ increase with depth and $\bar{k}(x)$ decreases.

The dependence of Λ_e on optical depth found for this case is closely approximated by Eq. (8).

We determine the constants $a = \bar{k}^*(0)$ and $b = \bar{k}^*(0) + s^*(0)$ for a layer of unit thickness by averaging $\bar{k}_\lambda$ and s_λ over the spectral composition of the quantities $E_0(0)$ and $q(0)$, which are determined at $x = 0$ from condition (5) by means of data for R and T of the irradiated layer of material. The constants a and b also provide an opportunity for calculating R_∞, T, and R from Eqs. (18), (22), and (23) and the radiation field $E_0(\tau)$, $q(\tau)$ from Eqs. (16)-(21), (24), and (25) in a layer of given material with varying optical properties.

With the help of experimental data [5] at x = 0, $\bar{k}^*(0) = 0.641$, and $s^*(0) = 3.862$, known for potato starch irradiated with KG-1000 lamps, an integral value $R_\infty = 0.612$ was obtained from Eq. (18) which differs by a total of 0.003 (an error of ~0.5%) from the integral reflectivity $R_\infty = 0.615$ obtained in [5] by integration of $R_{\infty,\lambda}$ over the entire spectrum.

To evaluate the accuracy of the analytic expressions obtained which characterize radiation flux distributions in materials with varying coefficients $\bar{k}(x)$ and $s(x)$ (see Fig. 2a), the quantities $E_0(x)$ and $q(x)$ were calculated from Eqs. (24) and (25) for irradiation of a starch layer by the integrated radiation flux from KG-1000 lamps.

It is clear from Fig. 2b that at the layer boundary x = 0 and at various values of x, the values of E_0^* and q^* obtained by integration over the spectrum and calculated from the derived analytic expressions (24) and (25) differ on the average by 1% which indicates rather high accuracy of the method proposed for the solution of the problem of radiation energy transport in absorbing and scattering materials with optical properties varying along a coordinate.

NOTATION

k, σ, absorption and scattering coefficients of an elementary layer of material, m^{-1}; $\Lambda = \sigma/(k+\sigma)$, photon survival probability, scattering criterion; $\bar{k}$, s, ε_e, averaged coefficients of absorption, backscattering, and extinction of an elementary layer, m^{-1}; $\Lambda_e = s/(\bar{k}+s)$, mean effective photon survival probability, Schuster criterion; τ, optical depth; $\varkappa$, γ, constant coefficients; β, ν, real or imaginary numbers; E, radiation flux density, W/m^2; E_0, spatial irradiance, W/m^2; q, resultant flux density, W/m^2; R, T, reflectivity and transmittance of a layer of finite thickness l; R_∞, reflectivity of an optically infinitely thick layer; λ, wavelength, μm. Indices: λ, spectral; i, incident; e, effective.

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HEAT EXCHANGE AND FRICTION IN A SUBSONIC VAPOR FLUX OF HIGH-TEMPERATURE HEAT PIPES

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The influence of forced vapor convection on heat transport in heat pipes is examined on the basis of the solution of the energy and motion equations. It is shown that radial heat flux due to molecular heat conduction of the vapor in the evaporator is negligible.

High-temperature heat pipes are ordinarily characterized in the literature as isothermal apparatuses. However, depending on the heat-exchange conditions in the surrounding medium and the magnitude of the power being transmitted, modes can exist where the axial temperature profile is characterized by abrupt changes from the maximum value at the beginning of the evaporator to the temperature of the surrounding medium at the end of the condenser. C. A. Busse gave a demarcation of heat pipe operating modes and typical axial temperature profiles. Taken as the viscous flow mode is that for which the vapor pressure at the end of the condensation zone is approximately equal to the vapor pressure at the temperature of the

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